Collective Neutrino Oscillations

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Outline

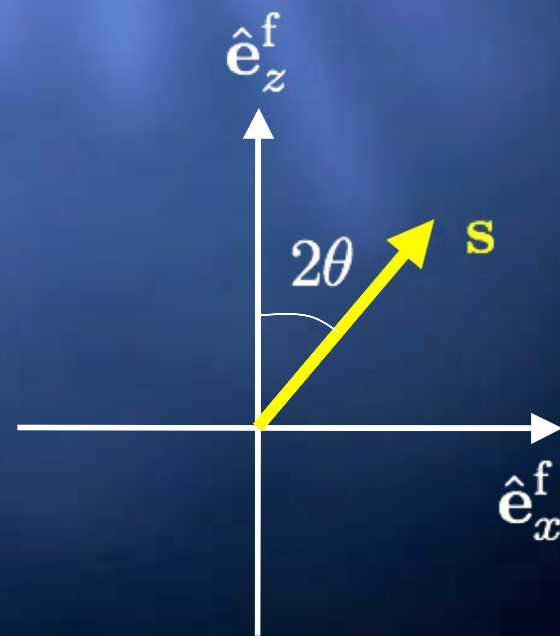
- Neutrino oscillation in the spin language.
- Simple collective neutrino oscillations.
- Summary.
- Collective neutrino oscillations in supernovae.

Spin Language

Flavor IsoSpin

$$\psi_{\nu}^{\mathrm{f}} \equiv \begin{pmatrix} a_{\nu_e} \\ a_{\nu_\tau} \end{pmatrix} \qquad \psi_{\bar{\nu}}^{\mathrm{f}} \equiv \begin{pmatrix} -a_{\bar{\nu}_\tau} \\ a_{\bar{\nu}_e} \end{pmatrix}$$

$$\mathbf{s} \equiv \psi^{\dagger} \frac{\boldsymbol{\sigma}}{2} \psi$$



Vector Fields

Generally, $\mathcal{H} = -\frac{\mu}{2}(H_0 + \mathbf{H} \cdot \boldsymbol{\sigma})$

For neutrino mixing, usually there are more than one fields, which fall into two categories:

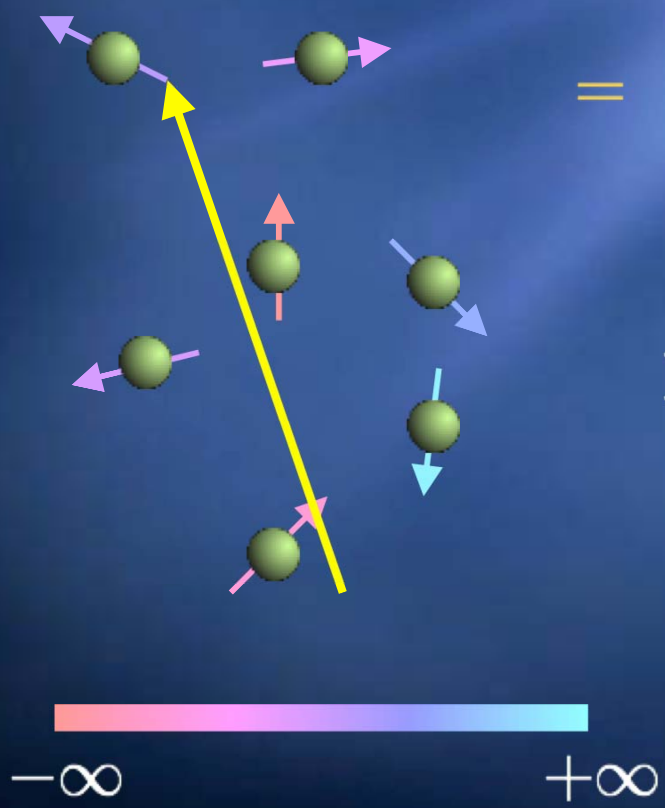
- those with μ's depending on the energy of the neutrino, and
- those with energy independent μ's.

Spin Language

Vector Fields

$$\mathcal{H}_{\mathrm{V}} = \frac{\delta m^2}{4E}\begin{pmatrix} -\cos 2\theta_{\mathrm{v}} & \sin 2\theta_{\mathrm{v}} \\ \sin 2\theta_{\mathrm{v}} & \cos 2\theta_{\mathrm{v}} \end{pmatrix}$$

$$= -\mu_{\mathrm{V}} \mathbf{H}_{\mathrm{V}} \cdot \frac{\boldsymbol{\sigma}}{2}$$



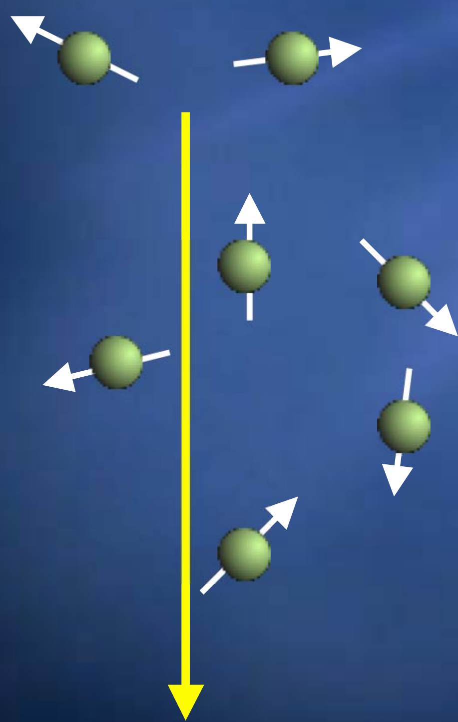
$$\mathbf{H}_{\mathrm{V}} \equiv -\hat{\mathbf{e}}_x^{\mathrm{f}} \sin 2\theta_{\mathrm{v}} + \hat{\mathbf{e}}_z^{\mathrm{f}} \cos 2\theta_{\mathrm{v}}$$

$$\mu_{\mathrm{V}} \equiv \frac{\delta m^2}{2E_\nu}, \text{ or } -\frac{\delta m^2}{2E_{\bar{\nu}}}$$

Spin Language

Vector Fields

$$\mathcal{H}_{\rm A} = \sqrt{2} G_{\rm F} n_e \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
$$= -\mu_{\rm A} \mathbf{H}_{\rm A} \cdot \frac{\boldsymbol{\sigma}}{2}$$



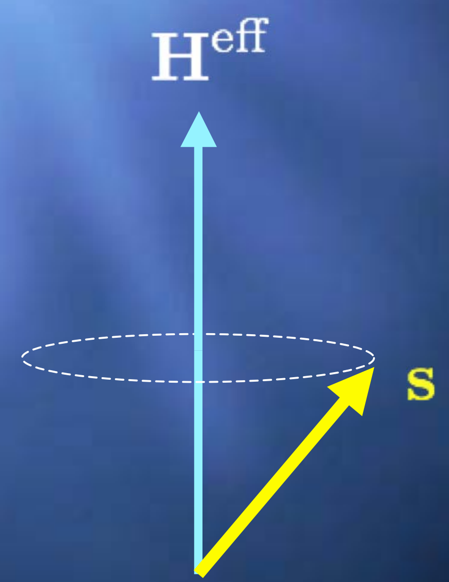
$$\mathbf{H}_{\rm A} \equiv -\hat{\mathbf{e}}^{\rm f}_z n_e$$

$$\mu_{\rm A} \equiv \sqrt{2} G_{\rm F}$$

Equations of Motion

$$\mathbf{H}^{\mathrm{eff}} \equiv \mu_{\mathrm{V}} \mathbf{H}_{\mathrm{V}} + \mu_{\mathrm{A}} \mathbf{H}_{\mathrm{A}} + \cdots$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{s} = \mathbf{s} \times \mathbf{H}^{\mathrm{eff}}$$



Equations of Motion

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{s}_i = \mathbf{s}_i \times (\mathbf{H}_i^{\mathrm{eff}} + \sum_j \mu_{ij} n_{\nu,j} \mathbf{s}_j)$$

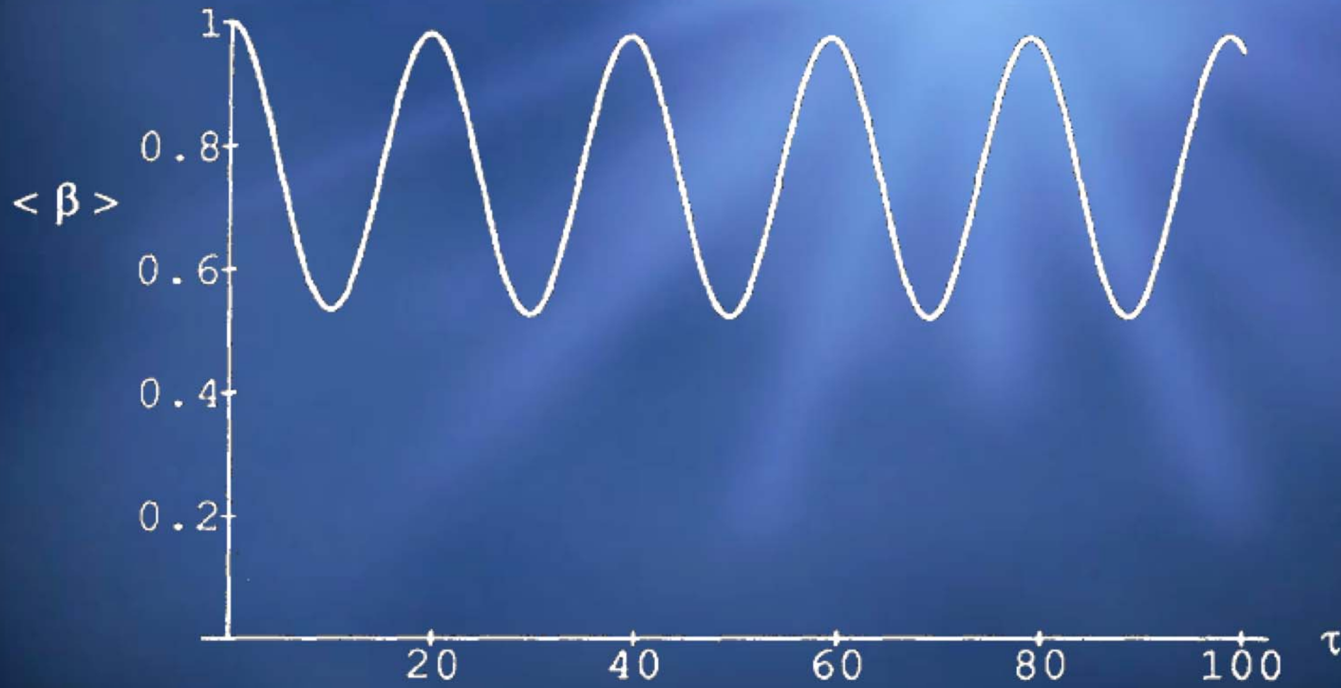
$$\mu_{ij} \equiv -2\sqrt{2} G_{\mathrm{F}} (1 - \cos\Theta_{ij})$$

Effective energy is conserved for static conditions.

$$\mathcal{E} \equiv -\sum_i n_{\nu,i} \mathbf{s}_i \cdot \mathbf{H}_i^{\mathrm{eff}} - \frac{1}{2} \sum_{ij} \mu_{ij} n_{\nu,i} n_{\nu,j} \mathbf{s}_i \cdot \mathbf{s}_j$$

Collective Oscillation

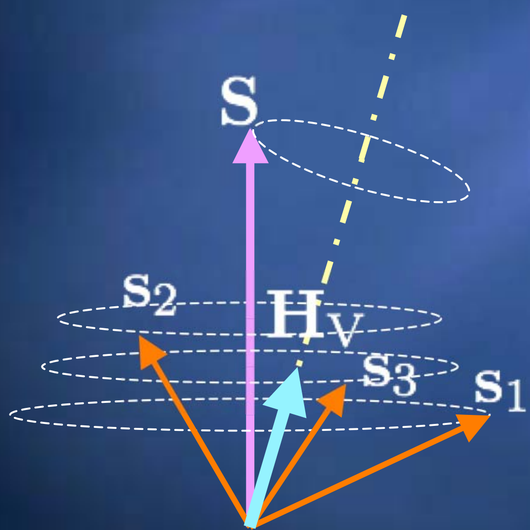
Synchronized System



S. Samuel, PRD 48, 1462 (1993)

Collective Oscillation

Synchronized System



$$\mu_{ij} \to \mu_\nu \equiv -2\sqrt{2}G_\mathrm{F}$$

$$\mathbf{S} \equiv \sum_i n_{\nu,i}\mathbf{s}_i$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{s}_i &= \mathbf{s}_i \times (\mu_{\mathrm{V},i}\mathbf{H}_\mathrm{V} + \mu_\nu \mathbf{S}) \\ &\simeq \mu_\nu \mathbf{s}_i \times \mathbf{S}\end{aligned}$$

$$\frac{\delta \mathbf{S}}{\delta t} \simeq \omega_\mathrm{sync}\mathbf{S} \times \mathbf{H}_\mathrm{V}$$

Synchronized System

$$
\begin{aligned}
\mathcal{E} &= -\sum_i \mu_{\mathrm{V},i} n_{\nu,i} \mathbf{s}_i \cdot \mathbf{H}_{\mathrm{V}} - \frac{\mu_\nu}{2}\mathbf{S}^2 \\
&\simeq -\frac{\mu_\nu}{2}\mathbf{S}^2
\end{aligned}
$$

Dense neutrino gas system will not transform from a coherent state (with large value of $|\mathbf{S}|$) into a completely incoherent state (with $\mathbf{S}=0$) or vice versa.

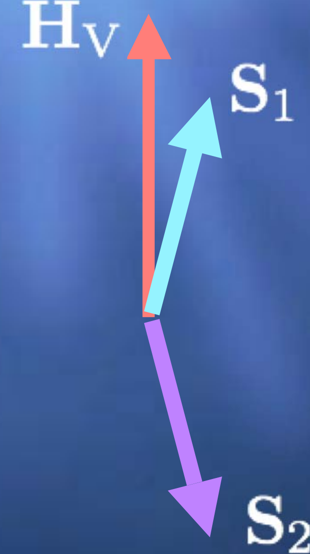
Bipolar System

Kostelecky & Samuel, Phys. Lett. B318, 127 (1993); PRD 52, 621 (1995)

Dense neutrino gases starting as ν_e and $\bar{\nu}_e$ experience some fast oscillations on timescales much shorter than that of vacuum oscillation. The configuration with an inverted neutrino mass hierarchy tend to have larger mixing than that with a normal mass hierarchy.

Bipolar System

$\mu_{\mathrm{V},1} = -\mu_{\mathrm{V},2} > 0 \ (\delta m^2 > 0)$, and $\mu_\nu < 0$



$$\mathbf{H}_\mathrm{V} \cdot (\mathbf{S}_1 + \mathbf{S}_2) = 0$$

$$\mathcal{E} = -\mu_{\mathrm{V},1}\mathbf{S}_1 \cdot \mathbf{H}_\mathrm{V} - \mu_{\mathrm{V},2}\mathbf{S}_2 \cdot \mathbf{H}_\mathrm{V} - \frac{\mu_\nu}{2}(\mathbf{S}_1 + \mathbf{S}_2)^2$$

Bipolar System

$-\mu_{\mathrm{V},1} = \mu_{\mathrm{V},2} > 0 \ (\delta m^2 < 0)$, and $\mu_\nu < 0$

$$\mathbf{H}_\mathrm{V} \cdot (\mathbf{S}_1 + \mathbf{S}_2) = 0$$

$$\mathcal{E} = -\mu_{\mathrm{V},1}\mathbf{S}_1 \cdot \mathbf{H}_\mathrm{V} - \mu_{\mathrm{V},2}\mathbf{S}_2 \cdot \mathbf{H}_\mathrm{V} - \frac{\mu_\nu}{2}(\mathbf{S}_1 + \mathbf{S}_2)^2$$

Bipolar System

- Collective oscillations on short timescales:

$$T_{\rm bi} \sim \sqrt{|\mu_{\rm V,1}\mu_\nu| n_\nu}$$
$$= \left(\sqrt{2}G_{\rm F} n_\nu \left|\frac{\delta m^2}{E_\nu}\right|\right)^{-1/2}$$

- Becomes synchronized if:

$$\frac{(n_{\nu,1} - n_{\nu,2})^2}{n_{\nu,1} + n_{\nu,2}} \gtrsim \left|\frac{\mu_{\rm V,1} - \mu_{\rm V,2}}{\mu_\nu}\right|$$

Summary

- Use Flavor IsoSpin to visualize neutrino oscillations.
- Interesting collective neutrino oscillations in dense neutrino gases.
- Where could collective neutrino oscillations occur?
  - Early Universe
  - Supernovae